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Coordinate-free characterization of the symmetry classes of elasticity tensors. (English summary)
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An elasticity tensor c_{ijkl} can be seen as a linear map c from the set of symmetric strain tensors ε_{kl} into the space of symmetric stress tensors σ_{ij}, such that $\sigma_{ij} = [c(\varepsilon)]_{ij} = c_{ijkl}\varepsilon_{kl}$ (with the convention of summation over repeated indices). Denote by an asterisk the left action of the group of rotations $\mathrm{SO}(3)$ on c_{ijkl} defined by $(A * c)(\varepsilon) = Ac(A^T \varepsilon A)A^T$ for each rotation A, and let $\mathcal{G}_c$ be the symmetry group of c, defined as

$$\mathcal{G}_c = \{A \in \mathrm{SO}(3)\colon\ A * c = c\}.$$

As proved by other authors [S. Forte and M. Vianello, J. Elasticity **43** (1996), no. 2, 81–108; MR1405284 (97h:73017)], the set of elasticity tensors can be subdivided into 8 symmetry classes, each one of them formed by all tensors whose symmetry groups are conjugate to each other. Thus, we have the symmetry classes of elasticity tensors which are classified as isotropic, transversely isotropic, cubic, orthotropic, etc. An interesting problem is the following: given an elasticity tensor c, whose Cartesian components are known with respect to some arbitrary frame of reference, find the symmetry class to which it belongs. This can be solved numerically but the goal of this paper is to propose an exact method, based on the eigendecomposition of c itself, seen as a linear map on the space of symmetric second-order tensors. It is shown that from a knowledge of (1) the eigenvalues of c and their multiplicity, (2) the eigendecomposition of the (symmetric) eigentensors which belong to the different eigenspaces of c, it is possible to fully determine the symmetry class of the given elasticity tensor c.

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